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Integrating Robustness into Multiobjective Space Vehicle Design Process

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The NASA Johnson Space Center employs Monte Carlo analysis incorporated within the Simulation and Optimization of Rocket Trajectories program to analyze trajectories of space vehicles. A basic function of this analysis is to assess the dispersion of a trajectory from a prescribed one due to dispersions of various vehicle and environmental parameters. The satellite design performance is evaluated using a large number of Monte Carlo simulations. Since this method is computationally very expensive, only a few design alternatives can be evaluated. In a previous paper, we argued that the computational efficiency of the design performance evaluation could be substantially improved by replacing the Monte Carlo simulations with a simulation technique based on orthogonal arrays. In this paper we have integrated this simulation technique into a multiobjective decision support environment using a compromise decision support problem formulation. This allows us to minimize the system variation while concurrently maximizing the achievement of other design goals, thus improving design quality by enabling a rational trade-off between nominal design performance and robustness.

Introduction

IN this paper we introduce a method for the design of a satellite trajectory as an example of the use of robust design in decision-based design (DBD). In an earlier paper,[1] we described how we could reduce the number of trajectory simulations necessary for determining the leading footprint of a space vehicle. This was achieved by replacing the Monte Carlo simulation with a simulation technique based on OAs. Here we go one step further by integrating these simulations into a multiobjective decision support environment using a DSP formulation. This design model allows us to minimize the system variation while concurrently maximizing the achievement of other design goals, thus improving design quality by enabling a rational trade-off between nominal design performance and robustness. The emphasis of this work is on the method rather than the results at per se.

A conceptual model for decision-based concurrent design was described earlier.[2] We offered DBD as a paradigm for the creation of design methods that are based on the notion that the principal role of a design engineer is to make decisions. We recognized that the implementation of DBD can take many forms, our implementation being the DSP technique. Support for human designers is provided through the formulation and solution of DSPs.[3]

In this paper we will first briefly recapture the Life Sat design problem[1,4] and the principles of robust design.[5] Then we will go into the details of the DSP formulation, taking both robustness and nominal design performances into consideration. We close the paper with a discussion of the results and future plans.

Life Sat Design Problem

Assume that a Life Sat vehicle and its trajectory are to be designed. A detailed description of the simulation model used to determine the landing footprint as a function of different design variables (control factors) and environmental parameters (noise factors) has been presented elsewhere,[3] the main control factors being vehicle mass, initial velocity, and flight-path angle. It is necessary to find the best value of these control factors to make the vehicle land on target (33.6 deg geodetic latitude) and to have the least variation around this target. It is also desirable to obtain a small value for the maximum acceleration in order to decrease vehicle heating due to drag. Ideally the acceleration should be about 50 m/s^2 and the maximum value should not be greater than 140 m/s^2. Lower and upper bounds on mass are 1400 and 1800 kg, respectively. The bounds on velocity are 9.8 and 10.2 km/s and the bounds on the flight-path angle are -6.2 and -5.5 deg. The initial geodetic latitude is held constant at 44.2 deg.

Introduction to Robust Design

Phadke,[5] following Taguchi, measures the quality of a product in terms of the total loss to society due to functional variation and harmful side effects. Under the ideal quality, the loss would be zero; the greater the loss, the lower the quality. A number of parameters (also known as factors) influence the quality characteristic Y of a system. Three types are distinguished: Signal factors M are set by the user or operator of the product to express the intended value for the product's response (target value). Noise factors x are parameters whose values cannot be deterministically predicted or controlled and cause the response Y to deviate from the target specified by the signal factor M, thus leading to a loss in quality. Levels of noise factors change from product to product, from one environment to another, and from time to time. Control factors z are parameters that can be specified by a designer so that system performance reaches the targeted values.

Taguchi has suggested using a signal-to-noise ratio η as a predictor of quality loss after making certain simple adjustments to the system's function.[5] Thus a set of observations is converted into a single number and is used as the objective function to be maximized in the robust design. Phadke and Taguchi recommend maximization of the signal-to-noise ratio:

$$\eta = \log\left(\frac{\bar{y}^2}{S^2}\right) \tag{1}$$

where $\bar{y}$ is the signal intensity, i.e., the mean value of the response, at the target value and S is the deviation. Here, η is expressed in decibels.

The quality characteristic for the Life Sat is its ability to follow a desired trajectory and land at a desired target. The response of a proposed design is its actual trajectory including deviations from

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the target. Noise factors are environmental and vehicle parameters. Control factors are related to the vehicle itself and are specified by the designer, e.g., mass, velocity, dimensions, and/or coefficients. In this case there are no signal factors. In the Life Sat example we are concerned with the distribution of the landing position within specified tolerances on the target. Following the robust design approach, we want to decrease the variation around the mean of the landing position and also minimize the bias between the mean and the target.

Compromise Decision Support Problem

Phadke[5] provides guidelines for selecting quality characteristics. He uses only one characteristic per problem, namely the signal-to-noise ratio η. We believe that it is difficult, if not impossible, to find one single metric for assessing the quality of a product or a process. In our opinion since there are multiple objectives to be satisfied in design, there must be multiple aspects to quality. Therefore we assert that quality loss is dependent on a number of quality characteristics with different degrees of importance. Quality involves trade-offs, and the desired quality cannot always be achieved. Related to our view is a discussion by Bras and Mistree,[6] and one by Otto and Antonsson[7] on the possibilities and drawbacks of the Taguchi method. They offer some extensions, e.g., the inclusion of design constraints. They also note that the Taguchi method is single-objective. The question is raised how trade-off should be handled, but they do not answer it. We show how trade-off is modeled, namely, through compromise DSPs[4,8] as explained in this section.

If there is an analytical equation relating the response to the noise and control factors, then we can use a Taylor series expansion to determine the mean and variance of the response as functions of the means and variances of the noise and control factors. Solution strategies to model quality in decision models are shown using a compromise DSP. However, if an analytical relationship does not exist, which is the case here, the values for mean and standard deviation need to be determined experimentally. This requires a different approach.

In order to obtain a mathematical formulation, it is necessary to derive the equations for η, the mean, the standard deviation, and the maximum acceleration. We propose a variation of Taguchi's method. Taguchi recommends the optimization of the logarithm of the signal-to-noise ratio, that is,

$$\eta = \log\left(\frac{\bar{y}^2}{S^2}\right) \qquad \text{or} \qquad \eta = \log \bar{y}^2 - \log S^2 \tag{2}$$

where $\bar{y}$ measures the mean of the signal and S^2 is equivalent to the mean squared deviation. This method has been criticized extensively by statisticians. However, we have circumvented the difficulties associated with the Taguchi formulation. Maximizing the signal and decreasing the noise are independent goals in our formulation. The priorities on the satisfaction of these and any additional goals required for specific problems are independent and are determined by a designer.

To calculate η for the geodetic latitude, it is necessary to perform simulations using orthogonal arrays. The value of η depends on the dispersions of nine noise factors: atmospheric density, vehicle mass, drag coefficient, initial velocity, flight-path angle, altitude, geodetic latitude, azimuth, and longitudinal initial position. Of these, three factors, vehicle mass, initial velocity, and flight-path angle are also control factor F_i, which means that their nominal value can be controlled by the designer.

The functional relationship between the signal-to-noise ratio for the geodetic latitude (SNGL) and the three control factors is defined by

$$\text{SNGL} = f_1(F_1, F_2, F_3) \tag{3}$$

From experience through previous studies, the target value for SNGL is chosen to be $T_{\text{SNGL}} = 15$ dB. Equation (3) is thus modeled as a goal in the compromise DSP by

$$\frac{\text{SNGL}}{15.0} + d_1^- - d_1^+ = 1.0 \tag{4}$$

where d_i^- and d_i^+ denote negative and positive deviations from the target value, respectively. Similar functional relationships exist for the mean and standard deviation. The mean of the geodetic latitude (MGL) is obtained by

$$\text{MGL} = f_2(F_1, F_2, F_3) \tag{5}$$

The target for the mean of the geodetic latitude is $T_{\text{MGL}} = 33.6$ deg; hence the corresponding goal for the compromise DSP can be expressed as

$$\frac{\text{MGL}}{33.6} + d_2^- - d_2^+ = 1.0 \tag{6}$$

The standard deviation geodetic latitude (SDGL), given by

$$\text{SDGL} = f_3(F_1, F_2, F_3) \tag{7}$$

has a desired target of zero; SDGL = 0.0. The goal for the compromise DSP is calculated from

$$\text{SDGL} + d_3^- - d_3^+ = 0.0 \tag{8}$$

The maximum acceleration (MAXA) is obtained as

$$\text{MAXA} = f_4(F_1, F_2, F_3) \tag{9}$$

Our desired target is a maximum acceleration of 50 m/s^2. This goal for the compromise DSP is calculated from

$$\frac{\text{MAXA}}{50.0} + d_4^- - d_4^+ = 1.0 \tag{10}$$

For reasons related to payload and integrity a constraint on the mean acceleration (MACC) is introduced. This constraint limits the maximum acceleration to a value of 140 m/s^2; therefore, we get

$$\text{MACC} = g_1(F_1, F_2, F_3) = 140.0 \tag{11}$$

The bounds on the system variables (control factors) are introduced in the problem statement. The preceding leads to the mathematical formulation of the problem as given in Fig. 1. The formulation in Fig. 1 has four goals in order to provide information about the effects of multiple quality characteristics of the process. To facilitate comparisons between goals, the goals are normalized. This normalization is performed by dividing goals by their associated targets, or vice versa, in order to obtain values for deviation variables ranging between zero and one. The mathematical formulation given in Fig. 1 is solved using the Adaptive Linear Programming (ALP) algorithm in DSIDES.[4] The pre-emptive (lexicographic minimum) deviation function Z is not explicitly specified; several deviation functions are discussed and each is associated with different design scenario. The lexicographic minimum concept is necessary for the solution of our problem and is defined as follows.[9]

Lexicographic minimum. Given an ordered array $\boldsymbol{f} = (f_1, f_2, \ldots, f_n)$ of nonnegative elements f_k, the solution given by $f^{(1)}$ is preferred to $f^{(2)}$ if

$$f_k^{(1)} < f_k^{(2)}$$

and all higher order elements (i.e., $f_1, \ldots, F_{k-1}$) are equal. If no other solution is preferred to $\boldsymbol{f}$, then $\boldsymbol{f}$ is the lexicographic minimum.

As an example, consider two solutions $\boldsymbol{f}^{(r)}$ and $\boldsymbol{f}^{(s)}$ where

$$f^{(r)} = (0, 10, 400, 56)$$

$$f^{(s)} = (0, 11, 12, 20)$$

In this example, $f^{(r)}$ is preferred to $\boldsymbol{f}^{(s)}$. The value 10 corresponding to $\boldsymbol{f}^{(r)}$ is smaller than the value 11 corresponding to $\boldsymbol{f}^{(s)}$. Once a preference has been established, all higher order elements are assumed to be equivalent. Hence the deviation function **Z** for the pre-emptive formulation is written as

$$\mathbf{Z} = [f_1(d^-, d^+), \ldots, f_k(d^-, d^+)]$$

Given

- A model of the Life Sat trajectory;
- Signal to noise ratio for geodetic latitude: SNGL
- Mean of geodetic latitude; MGL
- Standard deviation for geodetic latitude: SDGL
- Maximum acceleration; MAXA
- Target for signal to noise ratio ; $T_{SNGL} = 15.0$
- Target for mean of geodetic latitude; $T_{MGL} = 33.6$
- Target for standard deviation; $T_{SDGL} = 0.0$
- Target for max. acceleration; $T_{MAXA} = 50.0$
- Upper limit for max. acceleration; $MACC \leq 140.0$
- Constants;
  The initial latitude = 44.2 deg;
  The dispersions of noise factors;

Find

- The values for unknown elements:
  The value for initial velocity IV.
  The value for vehicle mass VM.
  The value for flight path angle FP.

Satisfy

- The Constraint
  $MACC / 140.0 \leq 1.0$
- The Signal to Noise Ratio Goal
  $SNGL / T_{SNGL} + d_1^- - d_1^+ = 1.0$
- The Mean on Target Goal
  $MGL / T_{MGL} + d_2^- - d_2^+ = 1.0$
- The Standard Deviation Goal
  $SDGL + d_3^- - d_3^+ = 0.0$
- The Maximum Acceleration Goal
  $MAXA / T_{MAXA} + d_4^- - d_4^+ = 1.0$
- The Bounds
  $9.8 \leq IV \leq 10.2$
  $1.4 \leq VM \leq 1.8$
  $-6.2 \leq FP \leq -5.5$

Minimize

- The Deviation Function
  $Z = \{ f_1(d^-, d^+), \ldots, f_k(d^-, d^+) \}$.

Fig. 1 Mathematical formulation of compromise DSP for robust design of the Life Sat trajectory.

Before exploring the model's behavior, another aspect of the design model must be discussed. In this section, Eqs. (3–11) contain functions (f_i, g_1), which are not explicit algebraic expressions of the control factors F_1, F_2, and F_3. Only six of the nine noise factors are assumed to be constants during the calculations. The other three are system variables or control factors, as stated earlier. These are the vehicle mass, the initial velocity, and the flight-path angle. Their dispersions are given as a percentage of their means; hence the dispersions change with the mean values. The calculations of these values are only estimates of the actual population. We show[4] that the estimations based on Monte Carlo and OA simulation have only small differences. All of the functions $f_1, \ldots, f_4$ are internally evaluated by the simulation model of OAs using 27 experiments. This is a nonlinear programming problem that may also be nondeterministic.

If a Monte Carlo simulation is used to evaluate function values, they will not change from run to run assuming that all inputs are the same and the random-number generator always starts with the same seed number. With different seed numbers the function output varies, even for the same input. This causes difficulties for the solution. Using Eq. (11), for example, where $g_1 \leq 140.0$, if g_1 is evaluated with different seed numbers, it may have an estimated value of 139.0 the first time and a value of 141.0 the second time, although the other input factors remain the same. Using OAs, all the experiments are designed in advance and therefore the function output is deterministic for a given input. Since the input variables are continuous, the output of functions $f_1 \ldots, f_4$ and g_1 is continuous and differentiable and the problem can be solved using the ALP algorithm. Gradients calculated in this algorithm are based on the results of the simulation with 27 experiments for a function evaluation. In the following section we explore the Life Sat design model. This provides information for the designer and helps to understand the model behavior.

Solving the Compromise DSP: Model Exploration

All nonlinear optimization algorithms are sensitive to the quality of the starting point. One optimization run can best identify one local

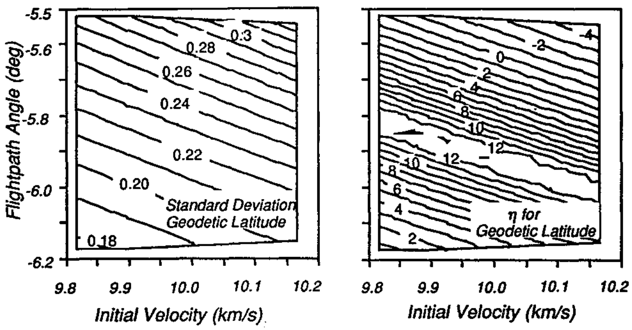

Fig. 2 Contour plots of initial velocity vs. standard deviation and η for geodetic latitude for constant mass.

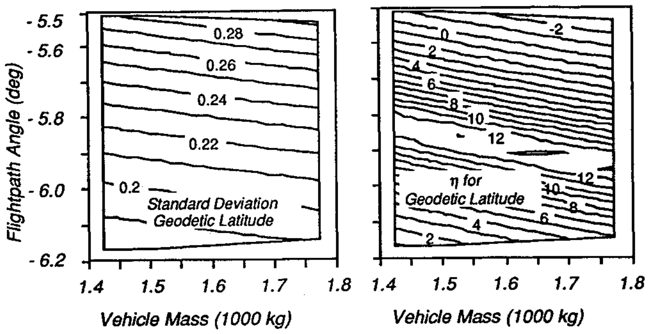

Fig. 3 Contour plots of vehicle mass vs standard deviation and η for geodetic latitude with constant initial velocity.

minimum. We generate contour plots in order to obtain a better understanding of the behavior of the Life Sat model. These plots also serve to validate the results obtained by solving the model using the ALP algorithm. Each contour plot consists of 200 equally distributed points generated by the XPLORE feature within the DSIDES system. XPLORE in DSIDES facilitates parametric studies of a design space. The time spent to evaluate these 200 points is approximately 10 min on a Sun 4/110 Sparcstation. Since we use the orthogonal array L_{27} for the simulation, a total of $200 \times 27 = 5400$ simulations must be performed. In Fig. 2 two contour plots of velocity versus flight-path angle are presented. The vehicle mass is kept constant at a value of $m = 1560$ kg. The contours represent values for the standard deviation and η.

There is a linear relationship between the three parameters (velocity, standard deviation, and flight-path angle), as shown on the left hand side of Fig. 2. For speeds below 9.9 km/s and a flight-path angle less than −6.1 deg, there is a very small standard deviation σ of approximately 0.16. For high values of speed and flight-path angle the standard deviation is approximately doubled. Although a small variation of the output is desirable, it is also important to reach the target, which is 33.6 deg of geodetic latitude. On the right-hand side of Fig. 2, the values for η provide this information. There is a band of high values, ranging from −6.0 to −5.8 deg for the flight-path angle and over the whole range of velocity. Thus, given that it is important to land on target, it is not possible to obtain a MSD smaller than 0.2 for this particular value of mass. In this region, η is greater than 12.0. When there are only two design variables, initial velocity and flight-path angle, a designer should choose mean values that lie within this band.

For the results shown in Fig. 3, the initial velocity is held constant at 9946.5 km/s and the vehicle mass and the flight-path angle are varied. The trends in Fig. 3 are similar to those in Fig. 2. There is a smaller range in the standard deviation that varies between 0.18 and 0.28. The relative difference between the lower and the upper value of the mass is larger (25% difference) compared to the relative difference in the velocity values (4% difference). The sensitivity of the standard deviation and η flight-path angle is larger for a particular value of mass, compared to the velocity.

Additional uses of the XPLORE feature for design space exploration are demonstrated elsewhere.[10] With it important information can be obtained within a short time. From Figs. 2 and 3, we identify ranges of values for two system variables for good designs, which will be useful as starting points for optimization. As seen in Figs. 2 and 3 there are several equally good solutions within the identified band and hence there are several different designs that satisfy the goals.

Formulation of Different Design Scenarios

In this section, several scenarios are investigated in order to solve the compromise DSP. That is, the sensitivity of the solution will change the order of priority for the four goals. We use the observations from this section to explain the results.

By changing priority levels, the behavior of the mathematical model for the compromise DSP is exercised for different priorities in the goals. First our highest priority is to get the mean of the geodetic latitude on target. If this is achieved we want to minimize the standard deviation. Finally the maximum acceleration should be minimized. The η is not used as it is contained in the other goals. In Table 1, we present four different scenarios, each characterized by a different priority order. For the first scenario, using the concept of lexicographic minimization as introduced earlier, the deviation function is written as

$$Z(d^-, d^+) = [(d_2^- + d_2^+), (d_3^- + d_3^+), (d_4^- + d_4^+)]$$

The deviation function is written similarly for the remaining scenarios. All scenarios and corresponding priorities are shown in Table 1. In the second scenario the η goal has the highest priority, presenting the quality characteristic for robust design. The acceleration goal is assigned to the second priority. In the third scenario, the priority order of mean and standard deviation goals have been changed. Scenario 4 represents the acceleration goal as having the greatest priority.

As can be seen in Table 1, every goal has been assigned the highest priority once, and therefore designs that satisfy each of the goals will be identified. In the following, the results are explained for each starting point and for all four scenarios. We present final values for the three designs variables and for four goals.

Discussion of Results

For solution verification, we use three different starting points for the three design variables, namely, initial velocity, vehicle mass, and flight-path angle. The first starting point is a point within the ranges of the design variables, the second and third points are points on the bounds of the variables. For each of the scenarios in Table 1, these three different starting points are used as given in Table 2. The results from scenario 1 are shown in Table 3; in this scenario, the mean value for the geodetic latitude is exactly on target. The standard deviation is minimized on the second priority level and a value of $\sigma = 0.215$ is obtained. Values for the design variables have changed slightly compared to their initial values.

For the second scenario, in which η has the highest priority, there are larger changes in the design variables. The speed is decreased and the mass increased. Although the mean value is not exactly on target as before, the standard deviation could be decreased and the η increased. These results driven by the η are slightly better. In the third scenario, the standard deviation is minimized with highest priority. This is achieved with a value of $\sigma = 0.193$, but the mean is far from the target. Therefore, η has a small value. If we are interested only in minimizing the variation of an output, this would be a good design. For the design variables, the velocity has increased and mass approaches the lower bound. The flight-path angle is exactly on the lower bound. Compared to the contour plot in Fig. 2, it should be possible to obtain values for the smallest standard deviation σ of approximately 0.18. Since the contour is very flat in this area, the convergence criteria for termination of the iterations are satisfied. Hence, tighter stopping criteria would lead to a better solution. In scenario 4, the maximum acceleration is minimized. This means we want to find a smaller value for the highest acceleration occurring during the flight. This goal is achieved within one iteration for all three starting points. Velocity and mass are going toward their upper bounds while the flight-path angle goes to the lower bound. Even starting on the opposite bounds leads to this solution. Therefore,

Table 1 Scenarios for design of Life Sat vehicle

Scenario	Priority 1	Priority 2	Priority 3
1	$d_2^- + d_2^+$	$d_3^- + d_3^+$	$d_4^- + d_4^+$
2	$d_1^- + d_1^+$	$d_4^- + d_4^+$	$d_2^- + d_2^+$
3	$d_3^- + d_3^+$	$d_2^- + d_2^+$	$d_4^- + d_4^+$
4	$d_4^- + d_4^+$	$d_2^- + d_2^+$	$d_3^- + d_3^+$

Table 2 Starting points for design variables

Parameter	Starting point 1	Starting point 2	Starting point 3
Velocity, m/s	9,950.0	10,200.0	9,800.0
Mass, kg	1,750.0	1,400.0	1,800.0
Flight-path angle, deg	−5.90	−6.20	−5.50

the value of 94.8 m/s^2 is the smallest value for the maximum acceleration obtained within the given bounds. This goal results in bad values for the other goals. We are far from the target and the standard deviation is extremely high. Hence the signal-to-noise ratio is very low.

In Table 4, the results are presented for starting point 2. The results for changing the goal priorities are approximately the same for all four scenarios. The major difference lies in the values for the mass for the first three scenarios. Having started at the lower bound, the final values are still close to it. In scenario 2, velocity is on the upper bound and the results are not as good as for scenario 1. The design point is farther away from the target and the standard deviation is higher.

In Table 5, the results for the third starting point are presented. As for starting point 1, good results are obtained for the goals but different values are obtained for the design variables. Although the mass in scenario 3 is on the upper bound, there is a low value for the standard deviation. According to the contour plots in figures 2 and 3 and the stopping criteria, this result is a solution, since the velocity is on its lower bound.

The conclusions for the different scenarios and starting points are as follows:

1) The two goals in scenario 1, getting the mean on target and minimizing the standard deviation in this order, result in approximately the same solutions as using a single goal of maximizing η (scenario 2).

2) There is no unique solution. For different starting points, equally good results for the goals are obtained, but the values for the design variables are different and hence there are different, but equivalent, designs.

3) The order of priority for the quality aspects of mean and standard deviation in scenario 1 is essential to obtain a good design solution. As demonstrated in scenario 3, the opposite order results in a poor design, as reflected in the value of η.

In the following, we discuss two figures representing values of the deviation function for the first two priorities (Fig. 4) and values for the design variables (Fig. 5) for each iteration. This is done for three different starting points and for scenario 1 as shown in Table 2. Priority levels 1 and 2 represent the quality characteristics in scenario 1. The highest priority is assigned to making the mean of the geodetic latitude land on target; the second priority is the standard deviation goal.

From Fig. 4, for each of the priority levels there is convergence to approximately the same solution for all three starting points. The number of iterations required for starting points 2 and 3 is four; for starting point 1, it is five. Priority level 1 goes to zero, that is, it is on target. On priority level 2, minimizing the standard deviation, a value around 0.215 (see Tables 4–6) is obtained. Convergence is achieved within four to five iterations, but the first iteration is close to the solution. As indicated earlier in this section, design variables do not reach the same solution values for different starting points. In Fig. 5, the complete range of the variables is used as scale.

The initial velocity has different solution values, but they are in the same region. The vehicle mass converges to different final

Table 3 Results with starting point 1

Parameter	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Velocity, m/s	9,939.0	9,830.0	10,044.1	10,200.0
Mass, kg	1,758.4	1,787.3	1,800.0	1,800.0
Flight-path angle, deg	−5.944	−5.891	−6.20	−5.50
Mean μ, deg	33.600	33.608	34.098	31.209
Standard deviation, σ	0.215	0.212	0.193	0.376
Signal-to-noise ratio, dB	13.18	13.29	5.422	−7.684
Acceleration, m/s^2	132.43	131.35	145.21[a]	94.81

[a]Constraint violation acceptable.

Table 4 Results with starting point 2

Parameter	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Velocity, m/s	9,952.1	10,000.0	10,020.0	10,200.0
Mass, kg	1,584.6	1,414.6	1,440.0	1,800.0
Flight-path angle, deg	−5.897	−5.984	−6.13	−5.50
Mean μ, deg	33.604	33.635	34.024	31.209
Standard deviation, σ	0.217	0.223	0.204	0.376
Signal-to-noise ratio, dB	13.081	12.76	6.512	−7.684
Acceleration, m/s^2	131.11	133.486	142.410[a]	94.81

[a]Constraint violation acceptable.

Table 5 Results with starting point 3

Parameter	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Velocity, m/s	9,981.3	9,800.0	9,800.0	10,200.0
Mass, kg	1,800.0	1,770.0	1,800.0	1,800.0
Flight-path angle, deg	−5.981	−5.856	−6.062	−5.50
Mean μ, deg	33.600	33.617	34.097	31.209
Standard deviation, σ	0.215	0.212	0.190	0.376
Signal-to-noise ratio, dB	13.18	13.26	5.457	−7.684
Acceleration, m/s^2	133.38	130.58	141.2[a]	94.81

[a]Constraint violation acceptable.

Table 6 Comparison of equivalent designs

Parameter	Design 1	Design 2	Design 3
Velocity, m/s	9939.0	9952.1	9981.3
Mass, kg	1758.4	1584.6	1800.0
Flight-path angle, deg	−5.944	−5.897	−5.981
Signal-to-noise ratio, dB	13.18	13.08	13.18

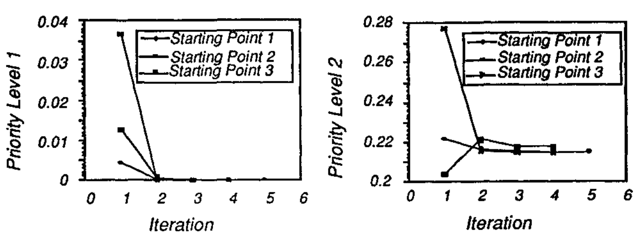

Fig. 4 Convergence of deviation function for two priority levels.

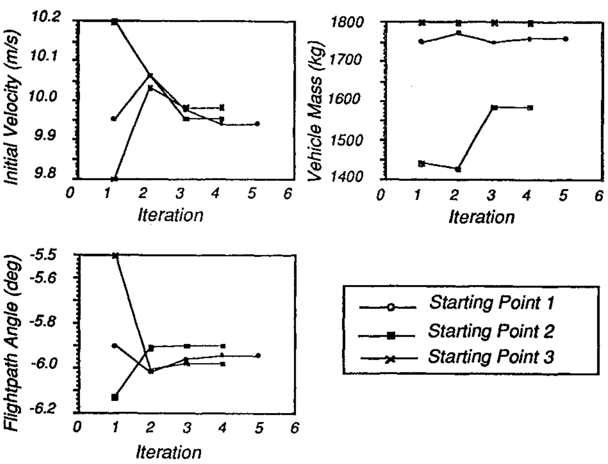

Fig. 5 Convergence of design variables for three starting points.

values covering a large range within the bounds. For starting point 3, the mass remains constant at the initial value. Only velocity and flight-path angle vary in this case, but a good design is obtained. In Table 6, three different but equivalent designs are obtained. This is reflected in similar values for η.

Figures 4 and 5 represent only scenario 1. For all other scenarios approximately the same results are obtained in the sense that only a few iterations are required for convergence but different solutions are obtained for different starting points. This is exactly what is expected from the contour plots, which indicate a band with equally high values for η. We could obtain these values with many different combinations of design variables.

Orthogonal array-based simulation is also much faster than the Monte Carlo simulations. In the following, we present the average time spent to simulate (on a SUN 4/110 Sparcstation) the trajectory using the robust design formulation of the compromise DSP: 1) 2–3 s for 27 simulations of the trajectory, 2) 3–5 min to perform a complete robust design study with three to four system variables using the compromise DSP, and 3) 10–15 min for design space exploration to detect best candidate starting points for optimization and information about the design space. When the trajectory is simulated with Monte Carlo simulation, the same robust design study takes approximately 3.5 h as compared to 3–5 min. We show that using the information obtained from 27 simulations based on OAs requires 40 times fewer simulations than the Monte Carlo simulation technique. The estimated time savings are approximately 97.5%.

Conclusions

In this paper we have shown the method for formulating and solving a compromise DSP for robust design, in this case the design of a satellite trajectory. The following has been illustrated:

1) The signal-to-noise ratio has been used to measure quantitatively the quality and the robustness of a design. The more robust a design, the less sensitive it is to noise factors. The goal is to find dimensions of design variables that give the most robust design.

2) The design space for the standard deviation and η is explored. Observations made from contour plots are helpful to explain and validate results. Design space exploration is also useful when dealing with highly nonlinear problems. Many conclusions can be drawn from contour plots obtained in a small amount of time (10–15 min for 200 points).

3) The Life Sat vehicle model has multiple acceptable solutions. Some of these solutions are identified within a few iterations. The time spent is approximately 2–3 min to obtain a satisfactory solution.

4) Only by replacing Monte Carlo simulations with a more efficient method, e.g., OAs,[4] are we able to solve effectively a design model such as the compromise DSP. We required only approximately 2.5% of the original computer time, or in other words, we can evaluate about 35–40 different designs in the time it takes to do one Monte Carlo simulation.

5) We do note that the use of OAs for system simulation and analysis has some limitation, leading in some cases to a reduced accuracy of the response estimation. This is especially so when the control factors exhibit strong interactions. However, we believe that for most real-world engineering design problems the benefit of being able to consider a large number of design alternatives represents a favorable trade-off. However, the final design solution should be verified using a control experiment, for instance by Monte Carlo simulation.

Future Work

Our goal is to provide the scientific foundation and develop a prototypical tool that can be used by human designers to determine concurrently the top-level specifications, the processes of design implementation, and the design of open engineering systems. We are investigating the use of OAs and robotus design technique in a concept exploration model. Traditionally, a concept exploration model is used by a designer to identify different concepts that are candidates for fulfilling the given top-level specifications. A concept exploration model can be used to quickly generate a large number of alternatives on a computer.[11]

We recognize that specifications lead to products. We propose the implementation of the compromise DSP as a bridge between conceptual and preliminary design of open engineering systems. To make a long-term, cost-effective contribution, we need to take into account designing for continuous improvements in order to maintain flexibility and concurrent engineering in order to address issues of design, manufacture, supportability, and the like.

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